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# Entropy Production, Information Generation and Evolution in the Expanding Universe

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## Entropy Production, Information Generation and Evolution in the Expanding Universe

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### Abstract

Information generating processes in the expanding universe are discussed on the basis of laboratory physics, in which the cosmic expansion was treated as the boundary conditions. Two important processes are shown to operate; dropping-out of the changing constraints due to the cosmic expansion, and gravothermal catastrophe of self-gravitating subsystems such as the stars. The cosmic expansion and resultant decoupling of gas from radiation is a prerequisite or a necessary condition for initiating the gravothermal catastrophe. Such approach reveals a clear understanding of the reasons why the universe is out of thermal equilibrium and why diversities are being generated in the universe rather than approaching towards the thermal death.

### §1. Introduction

According to Layzer<sup>1)</sup>, four classes of physical processes distinguish between the directions of the past and the future; (a) entropy-generating processes at the macroscopic and cosmological levels; (b) information-generating processes in open macroscopic systems and in the universe as a whole; (c) the cosmic expansion; and (d) the decay of neutral Kaons. He postulated *the strong cosmological principle* that there was no microscopic information in the initial state of the universe, and then he showed that the directions of the first three

classes of physical processes *could* coincide.

Rather than postulating such new principle, we will, here, follow another approach to the problem. We define the direction of the arrow of time by common sense or by the process (d). The recession of galaxies in the universe was measured in reference to this direction and it was referred to as the cosmic expansion or the expanding universe. Therefore, the expansion had been recognized only *after* the direction of the arrow was defined. (If defined otherwise, it would have been inferred to as contraction instead of expansion.) It is clear that the direction of the process (c) is the same as that of (d) by *definition*.

As far as a closed system in laboratory is concerned, its coarse grained entropy, i.e., the entropy at the macroscopic level, is well known to increase in time with the same direction as (d) when irreversible processes take place. (Here it is not our aim to discuss the reason why the coarse grained entropy does not decrease.) Even in the case of the universe, there is no reason to distinguish it from the laboratory. Let us consider a subsystem of cosmological scale which is expanding with the universe. When the cosmological principle, i.e., the homogeneous and isotropic universe, is postulated, this subsystem can be regarded to be isolated. If there should be mass- or energy-flux inflowing into the subsystem for example, this system could be distinguished from other part of the universe—violation of the cosmological principle. Therefore, the process (a) is also evident in the sense that the entropy is being produced in the universe by irreversible processes. (From the discussions above, it is also clear that entropy is produced even in the contracting universe if any deviation from thermal equilibrium exists.)

The most important process, which awaits for interpretation, is therefore the process (b), i.e., why the information-generating process is possible despite the second law of thermodynamics. We shall concentrate on this question in what follows.

After Layzer<sup>1)</sup> we define macroscopic information by

$$I \equiv (S_{\max} - S)/k \ln 2, \quad (1)$$

where  $S$  and  $k$  are the entropy of a system and the Boltzmann constant, respectively. When the system is in thermal equilibrium under given constraints, its entropy should be equal to its maximum possible value  $S_{\max}$ . Thus the information is measuring the deviation from the thermal equilibrium.

Considering that the system is initially in thermal equilibrium, (i.e.,  $I = 0$ , as assumed in the early stages of the expanding universe, but see also section 5c), we then ask how information can be generated. Since  $S$  is always non-decreasing, information can be generated only

when  $S_{\max}$  increases more rapidly than  $S$  (see also Layzer<sup>1)</sup>). There are two typical mechanisms. The first is the cosmic expansion, when it changes the constraints to the system more rapidly than relaxation by irreversible processes. In such cases thermal equilibrium state can not be recovered. An example is the element synthesis in the early stages of the expanding universe, as will be discussed in the next section.

The second is the mechanism which lifts off the thermal equilibrium itself, i.e., which makes  $S_{\max}$  infinitely large. Such mechanism is operative in the gravothermal catastrophe<sup>2)</sup> (section 3) or in the evolution of the stars (sections 4 and 5), where self-gravitation is essentially important. In such cases the system splits into two subsystems, the self-gravitating body ( $g$ ) and the surrounding radiation field ( $r$ ). The entropies of these subsystems will be denoted by  $S_g$  and  $S_r$ , respectively. In many cases  $S_g$  decreases while the total entropy  $S \equiv S_g + S_r$  increases. In such cases another definition of macroscopic information seems desirable.

The final fate of the gravothermal catastrophe is the formation of a black hole. If we take it into account,  $S_{\max}$  is finite yet quite large. Its implication to cosmology will be discussed in section 5. Finally in section 6, conclusions are summarized and the situation in the contracting universe will be briefly discussed.

## §2. Dropping-out of the Cosmic Expansion

In the early stages of the expanding universe, the expansion is very slow not in absolute value of its time scale but in comparison with the time scale to establish thermal equilibrium, for example, between the radiation field and matter. In order to seek a possibility for the emergence of information, we have to pay attention to reactions for which the relaxation time is or becomes longer than the time scale of the cosmic expansion. An example is the nuclear reactions; their reaction time becomes longer and longer when the temperature drops below about  $10^{10}$  K.

Here, we shall compute the entropy change associated with the reactions,

$$2p + 2n \rightleftharpoons \alpha . \quad (2)$$

We assume the ideal gas and the black-body radiation-field, which are in equilibrium each other under the temperature  $T$ . We consider a system consisting of  $N$  barions which are enclosed in a volume  $V$  with an adiabatic wall.

Entropy per barion and in units of the Boltzmann constant, i.e., the non-dimensional entropy, will be denoted by  $\sigma$ . When the system consists of equal numbers of protons, neu-

Entropy Production, Information Generation and Evolution in the Expanding Universe  
trons and electrons (state  $p$ ), the non-dimensional entropy is given by

$$\sigma = \sigma_{p,\text{gas}} + \sigma_{p,\text{rad}} ,$$

$$\sigma_{p,\text{gas}} = \frac{3}{2} \ln \frac{2e^{5/2} (2\pi m_p k T_p)^{3/2}}{(N/2V)h^3} + \frac{1}{2} \ln (m_e/m_p)^{3/2} , \quad (3)$$

$$\sigma_{p,\text{rad}} = \frac{4a T_p^3}{3k(N/V)} ,$$

where  $T_p$  is the temperature, and  $m_e$  and  $m_p$  are electron and proton mass, respectively.

When the system consists of  $N/4$  helium nuclei and  $N/2$  electrons under the temperature  $T_\alpha$  (state  $\alpha$ ), the corresponding entropies are

$$\sigma_{\alpha,\text{gas}} = \frac{3}{4} \ln \frac{2e^{5/2} (2\pi m_p k T_\alpha)^{3/2}}{(N/2V)h^3} + \frac{1}{2} \ln (2m_e/m_p)^{3/2} ,$$

$$\sigma_{\alpha,\text{rad}} = \frac{4a T_\alpha^3}{3k(N/V)} . \quad (4)$$

Here and in Eq. (3), mass differences between a proton, a neutron and a fourth of an  $\alpha$ -particle are neglected.

We shall consider a virtual change from the state  $p$  to  $\alpha$ . If the volume remains constant during it, the condition that the system is enclosed by the adiabatic wall is expressed as

$$a T_\alpha^4 V + \frac{9}{8} Nk T_\alpha = a T_p^4 V + \frac{9}{4} Nk T_p + q N , \quad (5)$$

where  $q = 7.33$  MeV is the energy difference per barion between these two states. When the energy per barion of the radiation field is much dominant over this  $q$ -value and  $kT_p$ , this condition is approximated by

$$T_\alpha - T_p \simeq \frac{(9/8)NkT_p + qN}{4a T_p^3 V} \ll T_p . \quad (6)$$

Thus the virtual entropy production associated with the virtual change of state is expressed by

$$\Delta\sigma = \sigma_\alpha - \sigma_p \simeq \frac{q}{kT_p} + \frac{9}{8} + \frac{1}{2} \ln 2^{3/2} - \frac{3}{4} \ln \frac{2e^{5/2} (2\pi m_p k T_p)^{3/2}}{(N/2V)h^3} . \quad (7)$$

If we put appropriate cosmological values into  $N/V$ , we find that  $\Delta\sigma$  of Eq. (7) vanishes for  $T = T_{\text{eq}} = 4 \times 10^9 \text{ K}$ . The thermal equilibrium state is the state  $p$  for  $T \gg T_{\text{eq}}$ , while it is the state  $\alpha$  for  $T \ll T_{\text{eq}}$ .

In the expanding universe, the value of  $V$  does not remain constant but increases. Since the energy of the radiation is dominant over the energy of the gas, the temperature decreases as  $T_p \propto V^{-1/3}$  as a first approximation. When the temperature drops close to  $T_{\text{eq}}$ , composition of the gas should begin to change from the state  $p$  to  $\alpha$  in order to keep pace with the changing nuclear statistical equilibrium. At this time, however, the nuclear reactions are so slow that most of the protons and the neutrons drop out of the changing thermal equilibrium.

The situation was oversimplified in the discussions above. In the actual history of the universe, the ratio of protons to neutrons was not unity mainly because of beta decays of neutrons, and the dropping-out was not complete either. However, this dropping-out mechanism generated information of the order of  $\Delta\sigma/\ln 2 \simeq q/k T_{\text{eq}} \ln 2 \simeq 30$  bits per nucleon. It should be compared with the total entropy which is close to the entropy of the radiation field, i. e.,  $\sigma \simeq \sigma_{\text{p,rad}} \simeq 4 \times 10^8$ . Though the information or the entropy deficiency generated by the dropping-out is very small as compared with the entropy of the background radiation field, yet it is a plenty of information in the standard of laboratory physics.

### §3. Gravothermal Catastrophe as a Source of Information

After the formation of the elements, the universe expands further to lower densities, and even a relaxation time between the radiation field and the matter becomes longer. Though the cosmic expansion becomes slower and slower in units of the cosmic time, it becomes faster and faster if we measure it in units of the relaxation time. At last the radiation field becomes decoupled from the matter.

After this decoupling time, the gravitational contractions of the gas are thought to prevail and the galaxies and then the stars are formed. They can be understood in terms of the gravothermal catastrophe of self-gravitating gas system. For the gravothermal catastrophe see a paper by Hachisu, Nakada, Nomoto and Sugimoto<sup>2)</sup>, which will be referred to as HNNS, and papers quoted therein.

We shall consider a spherical subsystem in the universe, which is assumed to be enclosed by an adiabatic wall with a constant volume. We assume also that it is in hydrostatic equilibrium and that temperature distribution is isothermal. According to the importance of the

self-gravitation relative to the thermal energy, this system exhibits different natures. We measure its importance in terms of the density contrast between the center and a shell just inside the wall, i. e.,  $D \equiv \rho_c/\rho_1$ . When  $D$  is smaller than its critical value  $D_{cr} = 709$ , the system is in stable thermal equilibrium and will be called ordinary thermal system. When  $D > D_{cr}$ , it will be called a gravothermal system<sup>3)</sup> and is unstable against perturbation in temperature or specific entropy distribution; a temperature gradient emerges spontaneously from the isothermal state and the contraction or the expansion of the gas follows.

The gravothermal catastrophe proceeds as follows. If heat is transported inwards, the inner part of the system expands and the temperature thereof decreases below its initial level despite its gain of the heat. Then heat continues to flow inwards and the central part expands further. At last the density contrast becomes lower than the critical value and the system reaches another isothermal state of the ordinary thermal system. During this transition from one isothermal state to another, entropy is produced by the heat transport—irreversible process. As seen in figure 5 of HNNS, this entropy production is of the order of  $\Delta\sigma \approx 0.1$ .

More interesting is the opposite case, in which heat is transported outwards as a result of perturbation. The inner portion of the system contracts and the temperature thereof increases over its initial level despite the removal of heat. Then heat continues to flow outwards and the inner portion contracts further and further. As discussed by HNNS, there is no final state of this contraction, entropy of the system continues to increase indefinitely due to the entropy production by the heat conduction as seen in figure 5 of HNNS.

If a subsystem is formed initially in the gravothermal state, and if the initial perturbation induces the outward heat flow,  $S_{max}$  can be regarded to be infinitely large in the sense that there is no final state in thermal equilibrium. Thus, infinite information are to be generated by the potential gravothermal catastrophe. However, this is just a mathematical description of matter. How plenty of information is it, then? In order to answer it we have to discuss the fate of the catastrophe, i. e., the formation of a black hole.

#### §4. Stellar Evolution as a Gravothermal Catastrophe

##### 4a) Global thermodynamics of the stars

Before swallowed into a black hole, we shall discuss the evolution of the stars. In the usual theory of stellar structure and evolution, hydrostatic and gravitational equilibrium of the star is discussed separately from local thermodynamics of the stellar matter. Here, on the contrary, we shall discuss global thermodynamics of the stars.

Entropy of the star  $S_g$  is expressed by

$$S_g = \int_0^M s dM_r, \quad (8)$$

where  $s$  is the specific entropy,  $M_r$  is the Lagrangian mass coordinate for the interior of the star, and  $M$  is the mass of the star. The entropy  $S_g$  increases due to irreversible processes taking place inside the star, but decreases by emitting energy out of the star.

First of all, the entropy production due to the transport of heat<sup>4)</sup> is given by

$$\left( \frac{dS_g}{dt} \right)_{ht} = \int_0^M \left( \frac{\delta s}{\delta t} \right)_{ht} dM_r = \int_0^M \frac{L_r}{4\pi r^2 \rho} \frac{d}{dr} \left( \frac{1}{T} \right) dM_r, \quad (9)$$

where  $r$  is the spatial coordinate,  $\rho$  is the density, and  $L_r$  denotes the heat flux flowing out through a spherical shell of radius  $r$ . Using the relation  $dM_r/dr = 4\pi r^2 \rho$ , the extreme right hand side of Eq. (9) is integrated by parts and the boundary condition  $L_r(r=0) = 0$  is applied. Then we obtain

$$\left( \frac{dS_g}{dt} \right)_{ht} = \frac{L_{ph}}{T_{eff}} - \int_0^M \frac{1}{T} \frac{dL_r}{dM_r} dM_r, \quad (10)$$

where  $L_{ph}$  is the photon luminosity of the star and  $T_{eff}$  is the effective surface temperature of the star.

The energy conservation is expressed as

$$\frac{dL_r}{dM_r} = \epsilon_n + \epsilon_g, \quad (11)$$

where  $\epsilon_n$  is nuclear energy generation rate per unit mass. Neutrino loss is neglected for the sake of simplicity. (It could be easily included if we wish.) Usually,  $\epsilon_g$  is called *gravitational energy release*, but it is the rate of out-flow of heat energy from a mass element *which is induced by the gravitational contraction*. It is expressed as

$$\epsilon_g \equiv - \frac{dq}{dt} = -T \frac{ds}{dt} - \sum_k \mu_k \frac{dN_k}{dt}. \quad (12)$$

Here  $N_k$  and  $\mu_k$  are, respectively, the number of particles of the  $k$ -th kind which are contained in unit mass of matter, and its chemical potential per particle from which its restmass



energy is excluded.

Putting Eq. (11) into Eq. (10) and using Eq. (8), we find

$$\frac{dS_g}{dt} = \int_0^M \frac{\epsilon_n}{T} dM_r + \int_0^M \frac{(-1)}{T} \sum_k \mu_k \frac{dN_k}{dt} dM_r + \left( \frac{dS_g}{dt} \right)_{ht} - \frac{L_{ph}}{T_{eff}} \quad (13)$$

The first two terms in the right hand side are the entropy production due to the nuclear reactions. Note that the second term contributes positively when the compositions are approaching to those in nuclear statistical equilibrium. Of course, we can write down Eq. (13) directly by applying thermodynamics of irreversible processes to an open system. In this case, we can derive Eq. (11) conversely from Eq. (13).

#### 4b) Phase of nuclear burning

In the evolutionary phase of quasi-static nuclear burning, the entropy of the star  $S_g$  remains almost unchanged except for a small effect due to the change in  $N_k$ , i. e., due to the second term in Eq. (13). In such a steady state the entropy production by the heat transport is of the order of  $\sim L_{ph}/T_{eff}$ , but the entropy production due to the input of nuclear energy is of the order of  $L_{ph}/T_c$  where  $T_c$  is the temperature at the center of the star. The ratio of these two are  $T_c/T_{eff} \sim 10^3$ , i. e., the entropy production by the heat transport is most important.

Entropies thus produced is thrown off or, in other words, dumped into space with the photon luminosity at the rate of  $L_{ph}/T_{eff}$ . We may describe such a system as a stationary state eating negentropy at the rate of  $L_{ph}/T_{eff} - \int (\epsilon_n/T) dM_r$ . This is a similar state as those realized in living organisms<sup>5)</sup>. In the case of the living organisms, the ultimate source of negentropy is the high temperature photons, which come from the sun, i. e., from outside of the system. In the case of a star, it is the nuclear energy being released in its high temperature core, which is built in the system itself. The nuclear reactions proceed to the direction of realizing the nuclear statistical equilibrium for the given temperature, i. e., realizing the composition consisting mainly of iron. This process is regarded to be eating information which was produced by the mechanism of the dropping-out in the early stages of the expanding universe. However, all entropies produced by the star are dumped away into space.

#### 4c) Phase of gravitational contraction

When no nuclear reactions are taking place, the star is in the phase of gravitational con-

traction. In such a phase the entropy produced by the heat transport is simply dumped away together with the stellar radiation. The change in the total entropy of the star  $S_g$  is evaluated not by means of Eq. (13) but by integrating Eq. (11) together with Eq. (12), i.e.,

$$L_{\text{ph}} = \int_0^M \epsilon_g dM_r = - \int_0^M T \frac{ds}{dt} dM_r . \quad (14)$$

Unless the homology of the star and thus the functional form of the entropy distribution  $s(M_r)$  change greatly,  $S_g$  is decreasing as seen from Eq. (14). This implies that more entropies are dumped outside of the star than produced by the heat transport.

Except for degenerate stars such as white dwarfs and neutron star, this decrease in entropy is more than simple cooling, i.e., more than the cooling of a heated iron block, for example, which is placed in the air of a lower temperature. As the block is cooled, the temperature difference between the block and the surrounding air diminishes and the cooling becomes slower and slower. On the other hand, it is well known that the central temperature of the star rises as energy is radiated. Therefore, the dumping of entropy from the star continues indefinitely, until some other mechanism quenches it. This is the same process as the gravothermal catastrophe taking place in the core region.

#### 4d) Relation with the second law of thermodynamics.

In order to apply the second law of thermodynamics, a system should be enclosed with an adiabatic wall. Thus, we consider that the system consists of two subsystems, i. e., the space or the radiation field of volume  $V$  and the star embedded in this space. Entropies of these subsystems are denoted by  $S_r$  and  $S_g$ , respectively. The system is a gravothermal system as defined in the preceding section, since the density contrast between the center of the star and a point just inside of the wall is practically infinite. A star in the open space corresponds to the case of infinite value of  $V$ .

The total entropy  $S = S_r + S_g$  of this system increases in time, as the irreversible processes take place both interior to the star and in the outer space (see also section 6b). This is the same entropy increase as associated with the gravothermal catastrophe which was discussed in section 3, or as stated by the second law of thermodynamics.

If we see the subsystems more closely, this process is described as follows. Entropy is produced by the irreversible processes interior to the star. More entropies than this entropy production are dumped into space. As a result,  $S_g$  decreases, while  $S_r$  increases. The space

Entropy Production, Information Generation and Evolution in the Expanding Universe  
is a garbage of entropies.

As such gravitational contraction proceeds, the distributions in densities and temperatures deviate from homogeneity more and more, and the diversity—if we use terminology of the evolutionism—emerges.

## §5. Entropy Associated with Black Holes

### 5a) Entropy production in the formation of black holes

The fate of the gravothermal catastrophe<sup>3)</sup> and of the evolution of the stars is the formation of a black hole, unless the mass of the system is too small. Entropy and temperature of the black hole are defined in relation with the evaporation of black holes due to quantum effect in curved space-time<sup>6)</sup>. For simplicity we shall consider black holes of spherical symmetry without charges and angular momenta. Then, their entropy and temperature are expressed as

$$\sigma_{\text{BH}} \equiv \frac{S_{\text{BH}}}{kN} = \frac{4\pi}{N} \left( \frac{M_{\text{BH}}}{m_g} \right)^2 \simeq 8.8 \times 10^{19} \left( \frac{M_{\text{BH}}}{M_{\text{BH}}^*} \right) \left( \frac{M_{\text{BH}}}{M_{\odot}} \right) , \quad (15)$$

$$T_{\text{BH}} = \frac{m_g c^2}{8\pi k} \frac{m_g}{M_{\text{BH}}} \simeq 10^{-7} \left( \frac{M_{\odot}}{M_{\text{BH}}} \right) \text{ K} , \quad (16)$$

respectively. Here,  $m_g$  and  $N$  are, respectively, the planck mass

$$m_g = (\hbar c / G)^{1/2} \simeq 2 \times 10^{-5} m_p , \quad (17)$$

and the barion number which is related with the proper mass of the black hole  $M_{\text{BH}}^*$  by

$$N = M_{\text{BH}}^* / m_p . \quad (18)$$

As seen in Eq. (15), the entropy of a black hole is enormously large as compared with the entropies of the stars, of the dropped-out material and even of the cosmic background radiation. This implies that the general relativistic gravitational collapse is an extremely irreversible process. In other words, an inverse process to destruct the black hole is extremely difficult and requires enormous amount of negentropies. (For example, operate a huge electric refrigerator to cool down the surrounding space below the temperature of the black hole, and then wait until the black hole evaporates away.)

5b) Maximum possible entropy or information contained in the selfgravitating system

In sections 3 and 4a, the maximum possible entropy  $S_{\text{max}}$  of a selfgravitating system was regarded to be infinitely large, when it was in the course of the gravothermal catastrophe. Here, however, we have to replace the  $S_{\text{max}}$  with the entropy of the black hole. Nevertheless, this  $S_{\text{max}}$  is practically infinitely large as compared with the entropy of the system *before the formation of the black hole*.

Is it appropriate and productive to measure the macroscopic information contained in the selfgravitating system by means of Eq. (1) even in such a case? If it be granted, practically infinite information would have been generated, when the cosmic gas decoupled from the radiation field in the expanding universe and the gravothermal catastrophe became possible. Afterwards, the selfgravitating systems or the stars eat this information. It would be the *devolution* of the star degrading into the black hole. However, such definition of evolution/devolution seems to be out of common sense. We should recognize rather the evolution to be a deviation from homogeneity. A precise definition of evolution still remains open.

5c) Primordial source of information in the expanding universe

Nevertheless we may inquire where such a plenty of information originally came from. In order to clarify it we have to consider the *eventual* thermal equilibrium of a system which consists of black holes and the radiation field. Since the coalescence of multiple black holes is an entropy producing process<sup>6)</sup>, we shall consider that there is only one black hole in the system.

We assume that the total (black hole plus radiation field) energy  $E$ , and the volume  $V$  are given. We calculate the total (black hole plus radiation field) entropy of the system. Then, by maximizing it we obtain stable thermal equilibrium, and we find mass of the black hole and temperature of the radiation field which is equal to the temperature of the black hole. For details see discussion by Davies,<sup>6)</sup> for example.

There are two distinct thermal equilibrium states depending on the value of

$$y \equiv \left[ a \left( \frac{m_g c^2}{3\pi k} \right)^4 \frac{V}{E} \right]^{1/4} \frac{m_g c^2}{E} \quad (19)$$

when  $y > 1.43$ , the thermal equilibrium state with maximum entropy consists only of the radiation field. When  $y < 1.43$ , on the other hand, there are two states which correspond to local maxima of entropy. One is the state consisting of pure radiation with high temperature,

and the other is the state consisting of black hole plus radiation with low temperature. The latter case is divided further into two. The state consisting of radiation has higher entropy for  $y > 1.01$ , while the state consisting of black hole plus radiation has higher entropy for  $y < 1.01$ .

Let us put cosmological values into Eq. (19). For the radiation dominant and matter dominant eras we obtain

$$y = 1 \times 10^{-43} (10^9 \text{ K} / T_r)^2, \quad (20)$$

$$y = 1 \times 10^{-31} (10^{-29} \text{ g cm}^{-3} / \rho_m)^{1/4},$$

respectively, where  $T_r$  is the radiation temperature and  $\rho_m$  is the matter density. Anyhow, the value of  $y$  is much smaller than 1.01 throughout the history of our expanding universe. This implies that the state with the maximum entropy should consist of black hole plus radiation at very low temperature.

However, the existence of primordial helium indicates that there was the radiation dominant era with high temperature. This implies that the expanding universe began from a state extremely far from the *eventual* thermal equilibrium and that the information was *potentially* contained already in the initial state of the universe.

The reason why it is *potentially* so is as follows. The state consisting of pure radiation is also a *stable* thermal equilibrium state since its entropy lies at one of the local maxima. A shift from the state with pure radiation to the state with black hole plus radiation is impossible unless relaxed is the condition of thermal equilibrium between the black hole and the radiation field. The shift becomes possible only after the system has been divided into subsystems and the gravothermal catastrophes become operative. This is the reason why the mechanisms to extract or realize the potential information, i. e., the dropping-out and gravothermal catastrophe, are important and had to be discussed in sections 2 and 3.

## §6. Conclusion and Discussions

### 6a) Conclusion

Though a precise definition of evolution was not obtained, the two important mechanisms are shown to extract information which was *potentially* contained in the initial state of the expanding universe. The first is the dropping-out mechanism which was discussed in section 3. The second is the gravothermal catastrophe which was discussed in sections 4 and 5.

Though the gravothermal catastrophe has been clearly shown to occur only in the case of spherical gas system, it may prevail even among self-gravitating collision-free stellar system such as galaxies and star clusters.

In the first mechanism the cosmic expansion was essential. On the other hand, the cosmic expansion is not directly related with the second mechanism. In order for the gravothermal catastrophe to proceed, however, the surrounding space with a relatively low temperature is a pre-requisite or a necessary condition. Such a clean space, which is unpoluted with entropies, was generated by the cosmic expansion after the decoupling of the cosmic gas from the radiation. (Though we have not discussed, the decoupling itself is also one of the dropping-out mechanisms which generates information during the later cosmic expansion.) The importance of radiation absorbing space and its relation with the cosmic expansion have also been pointed out by Gold<sup>7)</sup>.

We have discussed the information-generating processes on the basis of the laboratory physics, treating the cosmic expansion as the constraints or the boundary conditions. It has not been necessary at all to postulate a hypothesis that the cosmic expansion should have something inherent to do with determining the arrow of time.

#### 6b) Discussions

Since our discussions have been based only on the laboratory physics, it is easily extended to other problems. We shall briefly discuss some of such applications.

To begin with, we shall derive a condition that the star can evolve, i. e., can contract gravitationally by dumping entropies. We consider that the star is embedded in the radiation field with the temperature  $T_r$ . For simplicity the following approximations are applied; i) thermodynamical relations for the ideal gas, and ii) polytropic stellar structure with the index  $n = 3/2$  which corresponds to an isentrope. The stellar mass  $M$ , radius  $R$ , the central pressure  $P_c$  and the central density  $\rho_c$  are related by

$$M^2 = \frac{1}{4\pi G^3} \frac{P_c^3}{\rho_c^4} \phi_1^2, \quad R^2 = \frac{1}{4\pi G} \frac{P_c}{\rho_c^2} \xi_1^2, \quad (21)$$

where  $\phi_1$  and  $\xi_1$  are non-dimensional mass and radius of the polytrope.<sup>8)</sup> The total energy of the star<sup>8)</sup>  $E_{\text{tot}}$  is given by

$$E_{\text{tot}} = -\frac{3}{7} \frac{GM^2}{R} \quad (\text{for } n = 3/2) . \quad (22)$$

Using Eqs. (21) and (22) and the thermodynamical relations, the change in the entropy of the star is calculated as

$$dS_g = - \frac{3kN}{2\mu} d \ln ( -E_{\text{tot}} ) , \quad (23)$$

where  $\mu$  and  $N$  are the mean molecular weight and the barion number of the star, respectively.

We assume that the volume of the space surrounding the star is large enough but that the space is enclosed by an adiabatic wall. The photons emitted from the star will eventually degrade down to the black body of temperature  $T_r$ . Then the change in the entropy of the radiation field is given by

$$dS_r = \frac{1}{k T_r} d ( -E_{\text{tot}} ) . \quad (24)$$

The condition that the star can evolve is given by

$$dS = dS_g + dS_r > 0 \quad \text{for} \quad dE_{\text{tot}} < 0 . \quad (25)$$

Using Eqs. (23) and (24), it is rewritten as

$$( -E_{\text{tot}} ) > \frac{3}{2} \frac{N}{\mu} kT_r . \quad (26)$$

Using Eqs. (21) and (22), and equation of state, it is rewritten again as

$$T_c > \frac{7}{2} \frac{\xi_1}{\phi_1} T_r = 1.88 T_r , \quad (27)$$

where  $T_c$  is the temperature at the center of the star.

In the expanding universe the condition (27) is always satisfied after the decoupling of gas from radiation. In the contracting phase of a closed universe, the condition (27) will be eventually violated. In such situation the star will not contract gravitationally, but will dissolve absorbing the cosmic background radiation. This, however, does not imply that the arrow of time should be reversed as worried about by Gold.<sup>7)</sup> The total entropy of the star is increasing, i. e.,  $dS > 0$  for  $dE_{\text{tot}} > 0$ , along the *same* arrow of time. As seen in this example, our approach allows a simple picture for the arrow of time.

In the contracting universe every diversity dissolves both by the increase in  $S$  and by

the decrease in  $S_{\max}$ .<sup>9)</sup> However, the black holes will not be dissolved into thermal equilibrium. This is understood from the following three points. First of all the eventual thermal equilibrium for relatively small volume consists of a black hole plus radiation as discussed in section 5c. Secondly, the black hole continues to absorb the background radiation because the temperature of the black hole remains lower than that of the radiation field. Thirdly the entropy of the black holes is much larger than entropy of the cosmic black body radiation, and there would be no garbages at all to dump such a huge amount of entropy even if the black hole should dissolve.

Finally we wish to give a comment concerning the thermal death of the universe that was imagined by Boltzmann and others. Situation is now clear. Consider a system of cosmological size, the boundary of which is expanding with the cosmic expansion. Boltzmann was right in the sense that entropy in this system increases in time due to the irreversible processes. However, Boltzmann was wrong in the sense that the increase in the entropy of this system *does not* result in a thermally died, homogeneous distribution of temperature and density. Information is generated in the expanding universe which contains selfgravitating systems.

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